Differential astroseismic study of seismic twins observed by CoRoT

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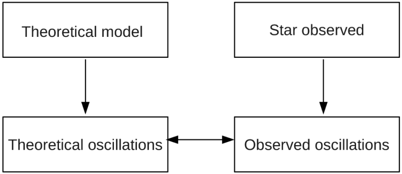
Introduction

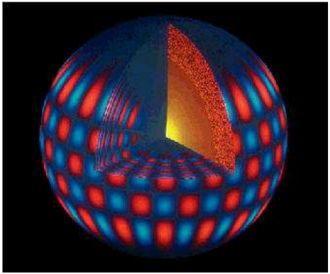
✓ Asteroseismology

✓ Seismic scaling relations

Asteroseismology

- ✓ description : study of stellar pulsations
- ✓ how does it work ?



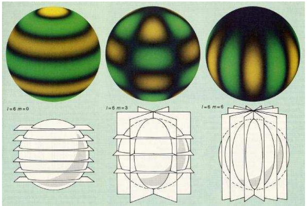


Stars in spherical symmetry

✓ Form of the solutions : $\Psi(r,\theta,\varphi) = \underbrace{\Psi_n(r)}_{\text{radial part}} \underbrace{Y_\ell^m(\theta,\varphi)}_{\text{spherical harmonics}}$

Quantum numbers

- ✓ n = radial order
- ✓ ℓ = degre ($\ell \geq 0$)
- ✓ m = azimutal order ($|m| \leq \ell$)



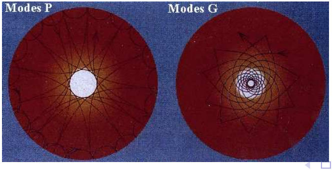
Different types of modes

Aqoustiques modes (p modes)

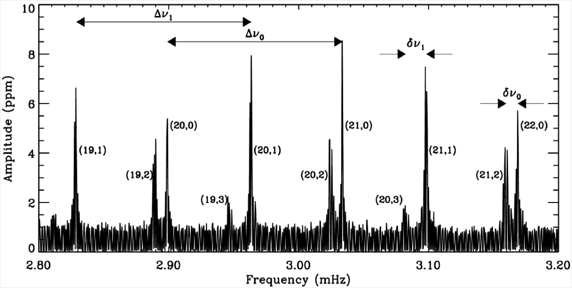
- ✓ the restoring force = the pressure
- ✓ frequencies are high and increases with $|n|$
- ✓ modes are concentrated at the surface

Gravity modes (g modes)

- ✓ the restoring force = the buoyancy (force of Archimède)
- ✓ frequencies are low and decreases with $|n|$
- ✓ modes are located in the interior of the stars



Diagnostic Potentials



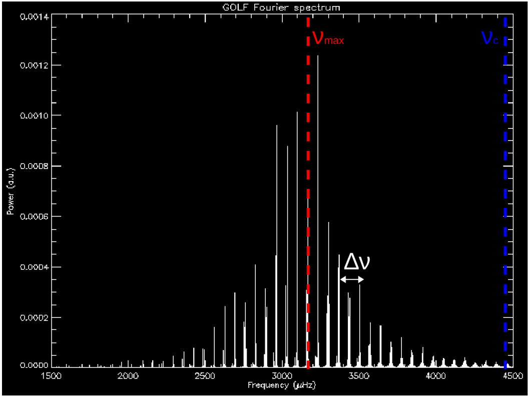
Solar Frequency Spectrum, as observed by VIRGO on SOHO satellite

Seismic indicators :

- ✓ **Large separation** : $\Delta_{n,\ell} = \nu_{n,\ell} - \nu_{n-1,\ell} \simeq \left(2\int_0^R \frac{dr}{c_s}\right)^{-1} \propto (M/R^3)^{1/2}$
- ✓ **Small separation** : $\delta_{n,\ell} = \nu_{n,\ell} - \nu_{n-1,\ell+2} \simeq -(4\ell+6)\frac{\Delta_{n,\ell}}{4\pi^2\nu_{n,\ell}}\int_0^R \frac{dc_S}{dr}\frac{dr}{r}$

un indicator of the age

Seismic indicators give global information about stars oscillations.



Chaplin et al. (2010)

- ✓ $\nu_{\rm max}$: Frequency of the maximum height in the power spectrum
- ✓ Δ_{ν} : Large separation

$\nu_{\rm c}$: Cut-off frequency

Scaling Relation

A seismic scaling relation :

A relation that relates global seismic indeces to fundamental stellar parameters.

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Mass, Radius, Effective temperature,...

Canonical Scaling Relations

✓ Ulrich (1986) showed that large seperation scales as the mean density in the context of solar-like pulsators.

$$\boxed{\Delta\nu \propto \rho^{1/2} \propto \left(\frac{M}{R^3}\right)}$$

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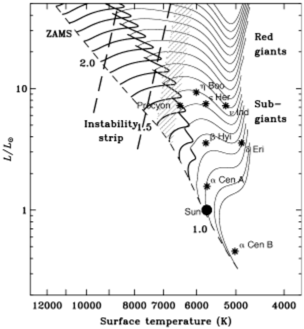
✓ Brown (1991) first proposed a linear relation between $\nu_{\rm max}$ and $\nu_{\rm c}$

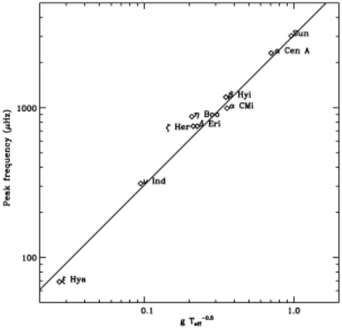
$$\nu_{\rm max} \propto \nu_{\rm c} \propto \frac{c_s}{2H_p} \propto \frac{g}{\sqrt{(T_{\rm eff})}} \propto \frac{M}{R^2\sqrt{(T_{\rm eff})}}$$

✓ This has been extended by Kjeldsen & Bedding (1995)

⇒ To predict mode amplitudes, frequency ranges, in solar-like stars

✓ Validated by Bedding & Kjeldsen (2003) using ground-based observations





(Bedding & Kjeldsen, 2003)

From $\Delta\nu$ and $\nu_{\max}$ to stellar masses and radii

- ✓ large seperation versus mean density $\Delta\nu \propto <\rho>^{1/2} \propto \left(\frac{M}{R^3}\right)^{1/2}$
- ✓ frequency of the maximum height versus cut-off frequecy $\nu_{\max} \propto \nu_c \propto \frac{c_s}{2H_p} \propto \frac{g}{\sqrt{(T_{\text{eff}})}} \propto \frac{M}{R^2\sqrt{(T_{\text{eff}})}}$

For a given effective temperature one can deduce an estimation of mass and radius.

$$R \propto \nu_{\max}\Delta\nu^{-2}T_{\text{eff}}^{1/2} \quad \Rightarrow \frac{R}{R_{\text{ref}}} = \left(\frac{\nu_{\max}}{\nu_{\max,\text{ref}}}\right)\left(\frac{\Delta\nu}{\Delta\nu_{\text{ref}}}\right)^{-2}\left(\frac{T_{\text{eff}}}{T_{\text{eff,ref}}}\right)^{1/2},$$

$$M \propto \nu_{\max}^3\Delta\nu^{-4}T_{\text{eff}}^{4/2} \quad \Rightarrow \frac{M}{M_{\text{ref}}} = \left(\frac{\nu_{\max}}{\nu_{\max,\text{ref}}}\right)^3\left(\frac{\Delta\nu}{\Delta\nu_{\text{ref}}}\right)^{-4}\left(\frac{T_{\text{eff}}}{T_{\text{eff,ref}}}\right)^{3/2}$$

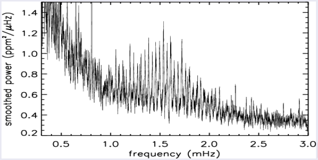
R and M ($\log g$) are often named seismic mass and radius (seismic gravity)

Plan

1. Introduction
   - Asteroseismology
   - Seismic scaling relations
2. Differential analysis method
   - STEP I : Finding a reference model
   - STEP II : Performing a differential analysis
3. Application to two CoRoT solar-like stars
   - Seismic Modelling of HD 181420
   - Differential Analysis for of HD 175272
4. Results
   - Results
5. Conclusion

Differential seismology of twins

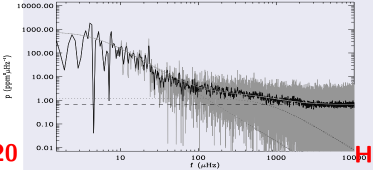
Space-based Observation : CoRoT,..



HD 181420

Barban et al. (2009)

Space-based Observation : CoRoT,..

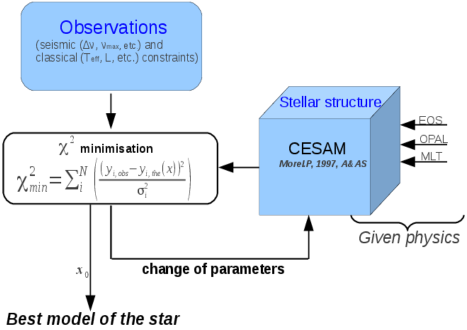


HD 175272

(Ozel et al. 2013)

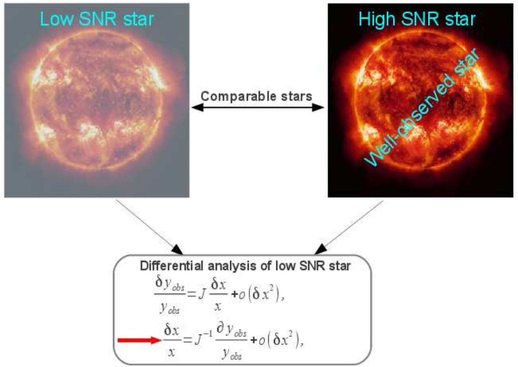
Schematic diagram : STEP I

New method : Find the best stellar model of the reference stars with a high SNR.



Schematic diagram : STEP II

New method : Perform the differential analysis to characterise an another star with a lower SNR.



where J is the Jacobien (∂lnyi/∂lnx), δyobs/yobs are the relative differences in observational constraints and δx/x are those in parameters between two stars.

HD 181420 and HD 175272

From HD 181420 with a high SNR to the less well-known star HD 175272

Table: Observations of HD 181420 and HD 175272 are determined by Barban et al. (2009), Mosser & Appourchaux (2009), respectively.

	HD 175272	HD 181420
$\Delta\nu$ (μHz)	74.9 ± 0.4	75.2±0.4
$\nu_{\rm max}$ (μHz)	1600±20	1590±10
$T_{\rm eff}$ (K)	6675±120	6580±100
[Fe/H]	+0.08 ± 0.11	−0.05 ± 0.06
$L/L_\odot$	6.3±1	4.28±0.28

STEP I : Seismic Modelling of HD 181420

Adopted the scaling relation :

$$\frac{\nu_{\max}}{\nu_{\max\odot}} = \frac{g}{g_\odot}\left(\frac{T_{\rm eff}}{T_{\rm eff\odot}}\right)^{-1/2}. \quad (1)$$

Table: Three different cases for modeling HD 181420

Case	Obs. Constraints	Model Parameters				Outputs	
		$M(M_\odot)$	$t($ Myr$)$	Y_0	α	$R/R_\odot$	$L/L_\odot$
I	$\Delta\nu$, $\nu_{\max}$	1.50	1000			1.62	6.26
II	$\Delta\nu$, $\nu_{\max}$, $T_{\rm eff}$	1.58	1470	0.20		1.69	5.24
III	$\Delta\nu$, $\nu_{\max}$, $T_{\rm eff}$, $L/L_\odot$	1.53	1460	0.19	1.05	1.66	4.44

STEP I : Seismic Modelling of HD 181420

Table: Model parameters and the theoretical values of the observational constraints are obtained using the Levenberg-Marquardt algorithm that searches the best-fit parameters by χ^2 minimisation.

	solar mixture GN93	solar mixture AGS05
M_1	1.30 ± 0.17	1.28 ± 0.17
t_1(Myr)	2127 ± 175	2325 ± 267
$(Y_0)_1$	0.30 ± 0.09	0.29 ± 0.09
$R/R_\odot$	1.61 ± 0.10	1.60 ± 0.10
$\Delta\nu_{\rm theo}(\mu$Hz)	75.2	75.2
$T_{{\rm eff}_{\rm theo}}$(K)	6542	6574
$L/L_{\odot_{\rm theo}}$	4.28	4.29

STEP II : Differential Analysis for of HD 175272

A first order Taylor development around the reference star gives, after some manipulation, the following linear system of equations :

$$\frac{\nu_{\max}}{\nu_{\max,\mathrm{ref}}} = \frac{g}{g_{\mathrm{ref}}} \left(\frac{T_{\mathrm{eff}}}{T_{\mathrm{eff,ref}}} \right)^{-1/2} \quad (2)$$

$$\frac{\mathrm{d}\nu_{\max}}{\nu_{\max}} - \frac{7}{2}\frac{\mathrm{d}T_{\mathrm{eff}}}{T_{\mathrm{eff}}} + \frac{\partial \ln L}{\partial \ln Z/X_0}\frac{\mathrm{d}Z/X_0}{Z/X_0} = \left(1 - \frac{\partial \ln L}{\partial \ln M}\right)\frac{\mathrm{d}M}{M} - \frac{\partial \ln L}{\partial \ln t}\frac{\mathrm{d}t}{t} - \frac{\partial \ln L}{\partial \ln Y_0}\frac{\mathrm{d}Y_0}{Y_0}, \quad (3)$$

$$\frac{\mathrm{d}T_{\mathrm{eff}}}{T_{\mathrm{eff}}} - \frac{\partial \ln T_{\mathrm{eff}}}{\partial \ln Z/X_0}\frac{\mathrm{d}Z/X_0}{Z/X_0} = \frac{\partial \ln T_{\mathrm{eff}}}{\partial \ln M}\frac{\mathrm{d}M}{M} + \frac{\partial \ln T_{\mathrm{eff}}}{\partial \ln t}\frac{\mathrm{d}t}{t} + \frac{\partial \ln T_{\mathrm{eff}}}{\partial \ln Y_0}\frac{\mathrm{d}Y_0}{Y_0}, \quad (4)$$

$$\frac{\mathrm{d}\Delta\nu}{\Delta\nu} - \frac{\partial \ln \Delta\nu}{\partial \ln Z/X_0}\frac{\mathrm{d}Z/X_0}{Z/X_0} = \frac{\partial \ln \Delta\nu}{\partial \ln M}\frac{\mathrm{d}M}{M} + \frac{\partial \ln \Delta\nu}{\partial \ln t}\frac{\mathrm{d}t}{t} + \frac{\partial \ln \Delta\nu}{\partial \ln Y_0}\frac{\mathrm{d}Y_0}{Y_0}, \quad (5)$$

where $\frac{\mathrm{d}M}{M}$, $\frac{\mathrm{d}t}{t}$, $\frac{\mathrm{d}Y_0}{Y_0}$ are the unknowns and $\frac{\mathrm{d}\nu_{\max}}{\nu_{\max}}$, $\frac{\mathrm{d}\Delta\nu}{\Delta\nu}$, $\frac{\mathrm{d}T_{\mathrm{eff}}}{T_{\mathrm{eff}}}$, $\frac{\mathrm{d}Z/X_0}{Z/X_0}$ are the seismic and non-seismic differential constraints.

Results

Table: Relative differences between the two stars, from Eqs. (3)-(5)

solar mixture	GN93	AGS05
Relative Differences of Observational Results		
$d\Delta\nu/\Delta\nu \pm \sigma_{\Delta\nu} = -0.004 \pm 0.007$		
$dT_{\mathrm{eff}}/T_{\mathrm{eff}} \pm \sigma_{T_{\mathrm{eff}}} = 0.014 \pm 0.023$		
$d\nu_{\mathrm{max}}/\nu_{\mathrm{max}} \pm \sigma_{\nu_{\mathrm{max}}} = 0.006 \pm 0.014$		
Relative Differences of Stellar Model Results		
$dM/M \pm \sigma_M$	0.06 ± 0.06	0.04 ± 0.05
$dt/t \pm \sigma_t$	-0.33 ± 0.26	-0.24 ± 0.27
$dY_0/Y_0 \pm \sigma_{Y_0}$	0.03 ± 0.12	0.07 ± 0.17
$dR/R \pm \sigma_R$	0.02 ± 0.02	0.01 ± 0.02

Results

Table: Parameters of HD 175272 obtained by adding the results of thefiggerential analysis with those obtained for HD 181420 for a full computation of adiabatic frequencies.

solar mixture	GN93	AGS05
Parameters of Models		
M_2	1.38±0.20	1.33± 0.36
t_2(Myr)	1521±271	1829± 245
$(Y_0)_2$	0.31±0.09	0.31± 0.16
Properties of Models		
$R_2/R_\odot$	1.64	1.65
$\Delta\nu$	75.01	72.80
$\nu_{\rm max}$	1455	1393
$T_{\rm eff}$	6655	6645
$L_2/L_\odot$	4.7	4.8
$\log g_2$	4.15	4.13

Conclusion

- ✓ The differential analysis method is based on scaling relations, and benefit from the comparison to a star with similar characterisitcs.
- ✓ The scientific output of many astroseismic objects with a low SNR benefit from the precise modeling of nearby reference stars with a high SNR.
- ✓ It can be applied to stars with interesting properties, such as stars hosting an exoplanet or members of a double system.

Perspectives

- Apply the same type of analysis to other types of CoRoT and Kepler's stars with a low SNR, from red giants to solar-like stars.
- Caracterize the well-constrained stars : a very precise determination of the structural differences between nearby stars.

THANKS FOR ATTENTION !