Differential astroseismic study of seismic twins observed by CoRoT

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19th National Astronomy Conference
METU, Ankara

February 02\textsuperscript{th} 2015
Introduction

✓ Asteroseismology

✓ Seismic scaling relations
Asteroseismology

✓ description: study of stellar pulsations
✓ how does it work?

Theoretical model → Theoretical oscillations → Star observed → Observed oscillations
Stars in spherical symmetry

✓ Form of the solutions: \( \Psi(r, \theta, \varphi) = \Psi_n(r) \ Y_{\ell}^m(\theta, \varphi) \)

Quantum numbers

✓ \( n = \) radial order
✓ \( \ell = \) degree (\( \ell \geq 0 \))
✓ \( m = \) azimuthal order (\(|m| \leq \ell\))
## Different types of modes

### Aqoustiques modes (p modes)
- ✓ the restoring force = the pressure
- ✓ frequencies are high and increases with $|n|$
- ✓ modes are concentrated at the surface

### Gravity modes (g modes)
- ✓ the restoring force = the buoyancy (force of Archimède)
- ✓ frequencies are low and decreases with $|n|$
- ✓ modes are located in the interior of the stars
Diagnostic Potentials

Solar Frequency Spectrum, as observed by VIRGO on SOHO satellite

Seismic indicators:

- **Large separation**: \( \Delta_{n,\ell} = \nu_{n,\ell} - \nu_{n-1,\ell} \approx \left(2 \int_0^R \frac{dr}{c_s} \right)^{-1} \propto \left(\frac{M}{R^3}\right)^{1/2} \)
- **Small separation**: \( \delta_{n,\ell} = \nu_{n,\ell} - \nu_{n-1,\ell+2} \approx -(4\ell + 6) \frac{\Delta_{n,\ell}}{4\pi^2 \nu_{n,\ell}} \int_0^R \frac{dc_s}{dr} \frac{dr}{r} \)

*un indicator of the age*
Seismic indicators give global information about stars oscillations.

\[ \nu_{\text{max}} : \text{Frequency of the maximum height in the power spectrum} \]

\[ \Delta \nu : \text{Large separation} \]

\[ \nu_c : \text{Cut-off frequency} \]
Scaling Relation

A seismic scaling relation:

A relation that relates global seismic indices to fundamental stellar parameters.
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A relation that relates global seismic indices to fundamental stellar parameters.

Mass, Radius, Effective temperature,...
Ulrich (1986) showed that large separation scales as the mean density in the context of solar-like pulsators.

\[ \Delta \nu \propto \rho^{1/2} \propto \left( \frac{M}{R^3} \right) \]
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\[ \Delta \nu \propto \rho^{1/2} \propto \left( \frac{M}{R^3} \right) \]

Brown (1991) first proposed a linear relation between \( \nu_{\text{max}} \) and \( \nu_c \)

\[ \nu_{\text{max}} \propto \nu_c \propto \frac{c_s}{2H_p} \propto \frac{g}{\sqrt{T_{\text{eff}}}} \propto \frac{M}{R^2 \sqrt{T_{\text{eff}}}} \]
✓ This has been extended by Kjeldsen & Bedding (1995) ⇒ To predict mode amplitudes, frequency ranges, in solar-like stars

✓ Validated by Bedding & Kjeldsen (2003) using ground-based observations

(Bedding & Kjeldsen, 2003)
From $\Delta\nu$ and $\nu_{\text{max}}$ to stellar masses and radii

- large separation versus mean density $\Delta\nu \propto <\rho>^{1/2} \propto (\frac{M}{R^3})^{1/2}$
- frequency of the maximum height versus cut-off frequency
  \[ \nu_{\text{max}} \propto \nu_{\text{c}} \propto \frac{c_s}{2H_p} \propto \frac{g}{\sqrt{T_{\text{eff}}}} \propto \frac{M}{R^2 \sqrt{T_{\text{eff}}}} \]

For a given effective temperature one can deduce an estimation of mass and radius.

\[
\frac{R}{R_{\text{ref}}} = \left( \frac{\nu_{\text{max}}}{\nu_{\text{max,ref}}} \right) \left( \frac{\Delta\nu}{\Delta\nu_{\text{ref}}} \right)^{-2} \left( \frac{T_{\text{eff}}}{T_{\text{eff,ref}}} \right)^{1/2},
\]

\[
\frac{M}{M_{\text{ref}}} = \left( \frac{\nu_{\text{max}}}{\nu_{\text{max,ref}}} \right)^3 \left( \frac{\Delta\nu}{\Delta\nu_{\text{ref}}} \right)^{-4} \left( \frac{T_{\text{eff}}}{T_{\text{eff,ref}}} \right)^{3/2}
\]

$R$ and $M$ (log $g$) are often named seismic mass and radius (seismic gravity)
Plan

1. Introduction
   - Asteroseismology
   - Seismic scaling relations

2. Differential analysis method
   - STEP I : Finding a reference model
   - STEP II : Performing a differential analysis

3. Application to two CoRoT solar-like stars
   - Seismic Modelling of HD 181420
   - Differential Analysis for of HD 175272

4. Results
   - Results

5. Conclusion
Differential seismology of twins

Space-based Observation: CoRoT...

Barban et al. (2009)

HD 181420

Space-based Observation: CoRoT...

(Ozel et al. 2013)

HD 175272
**Schematic diagram : STEP I**

**New method:** Find the best stellar model of the reference stars with a high SNR.

\[
\chi^2_{\text{min}} = \sum_{i}^{N} \left( \frac{y_{i,\text{obs}} - y_{i,\text{calc}}(x)}{\sigma_i} \right)^2
\]

- **Observations:** (seismic ($\Delta V$, $V_{\max}$, etc.) and classical ($T_{\text{eff}}$, $L$, etc.) constraints)
- **Stellar structure**: CESAM (Morel, 1997, A&A)
- **EOS**, **GPAI**, **MLT**
- **Given physics**
- **Best model of the star**
- **Change of parameters**
**Schematic diagram : STEP II**

**New method :** Perform the differential analysis to characterise an another star with a lower SNR.

\[
\frac{\delta y_{\text{obs}}}{y_{\text{obs}}} = \frac{\delta x}{x} + o(\delta x^2), \\
\frac{\delta x}{x} = J^{-1} \frac{\partial y_{\text{obs}}}{y_{\text{obs}}} + o(\delta x^2),
\]

where \( J \) is the Jacobian \( \frac{\partial y_{\text{obs}}}{\partial x} \), \( \delta y_{\text{obs}}/y_{\text{obs}} \) are the relative differences in observational constraints and \( \delta x/x \) are those in parameters between two stars.
From HD 181420 with a high SNR to the less well-known star HD 175272

**Table:** Observations of HD 181420 and HD 175272 are determined by Barban et al. (2009), Mosser & Appourchaux (2009), respectively.

<table>
<thead>
<tr>
<th></th>
<th>HD 175272</th>
<th>HD 181420</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \nu$ ($\mu$Hz)</td>
<td>74.9 ± 0.4</td>
<td>75.2 ± 0.4</td>
</tr>
<tr>
<td>$\nu_{\text{max}}$ ($\mu$Hz)</td>
<td>1600 ± 20</td>
<td>1590 ± 10</td>
</tr>
<tr>
<td>$T_{\text{eff}}$ (K)</td>
<td>6675 ± 120</td>
<td>6580 ± 100</td>
</tr>
<tr>
<td>[Fe/H]</td>
<td>+0.08 ± 0.11</td>
<td>−0.05 ± 0.06</td>
</tr>
<tr>
<td>$L/L_\odot$</td>
<td>6.3 ± 1</td>
<td>4.28 ± 0.28</td>
</tr>
</tbody>
</table>
STEP I : Seismic Modelling of HD 181420

Adopted the scaling relation:

\[
\frac{\nu_{\text{max}}}{\nu_{\text{max} \odot}} = \frac{g}{g_{\odot}} \left( \frac{T_{\text{eff}}}{T_{\text{eff} \odot}} \right)^{-1/2}.
\]

\( \text{(1)} \)

**Table:** Three different cases for modeling HD 181420

<table>
<thead>
<tr>
<th>Case</th>
<th>Obs. Constraints</th>
<th>Model Parameters</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>( \Delta \nu ), ( \nu_{\text{max}} )</td>
<td>1.50, 1000</td>
<td>1.62, 6.26</td>
</tr>
<tr>
<td>II</td>
<td>( \Delta \nu ), ( \nu_{\text{max}} ), ( T_{\text{eff}} )</td>
<td>1.58, 1470, 0.20</td>
<td>1.69, 5.24</td>
</tr>
<tr>
<td>III</td>
<td>( \Delta \nu ), ( \nu_{\text{max}} ), ( T_{\text{eff}} ), ( L/L_{\odot} )</td>
<td>1.53, 1460, 0.19</td>
<td>1.05, 1.66, 4.44</td>
</tr>
</tbody>
</table>

Nesibe OZEL
Differential Asteroseismology
STEP I : Seismic Modelling of HD 181420

**Table:** Model parameters and the theoretical values of the observational constraints are obtained using the Levenberg-Marquardt algorithm that searches the best-fit parameters by $\chi^2$ minimisation.

<table>
<thead>
<tr>
<th></th>
<th>solar mixture GN93</th>
<th>solar mixture AGS05</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>1.30 ± 0.17</td>
<td>1.28 ± 0.17</td>
</tr>
<tr>
<td>$t_1$ (Myr)</td>
<td>2127 ± 175</td>
<td>2325 ± 267</td>
</tr>
<tr>
<td>$(Y_0)_1$</td>
<td>0.30 ± 0.09</td>
<td>0.29 ± 0.09</td>
</tr>
<tr>
<td>$R/R_\odot$</td>
<td>1.61 ± 0.10</td>
<td>1.60 ± 0.10</td>
</tr>
<tr>
<td>$\Delta \nu_{\text{theo}}$ (µHz)</td>
<td>75.2</td>
<td>75.2</td>
</tr>
<tr>
<td>$T_{\text{eff, theo}}$ (K)</td>
<td>6542</td>
<td>6574</td>
</tr>
<tr>
<td>$L/L_\odot_{\text{theo}}$</td>
<td>4.28</td>
<td>4.29</td>
</tr>
</tbody>
</table>
STEP II : Differential Analysis for of HD 175272

A first order Taylor development around the reference star gives, after some manipulation, the following linear system of equations:

\[
\frac{\nu_{\text{max}}}{\nu_{\text{max,ref}}} = \frac{g}{g_{\text{ref}}} \left( \frac{T_{\text{eff}}}{T_{\text{eff,ref}}} \right)^{-1/2}
\]  

\[ \frac{\text{d} \nu_{\text{max}}}{\nu_{\text{max}}} \left( \begin{array}{c}
\frac{7}{2} \frac{\text{d} T_{\text{eff}}}{T_{\text{eff}}} + \frac{\partial \ln L}{\partial \ln Z/X_0} \frac{\text{d} Z/X_0}{Z/X_0} \\
\frac{\partial \ln T_{\text{eff}}}{\partial \ln Z/X_0} \frac{\text{d} Z/X_0}{Z/X_0} & \frac{\partial \ln \Delta \nu}{\partial \ln Z/X_0} \frac{\text{d} Z/X_0}{Z/X_0}
\end{array} \right) = \left( \begin{array}{c}
\frac{\partial \ln L}{\partial \ln M} \frac{\text{d} M}{M} - \frac{\partial \ln L}{\partial \ln t} \frac{\text{d} t}{t} - \frac{\partial \ln L}{\partial \ln Y_0} \frac{\text{d} Y_0}{Y_0}
\end{array} \right),
\]

\[ \frac{\text{d} T_{\text{eff}}}{T_{\text{eff}}} = \frac{\partial \ln T_{\text{eff}}}{\partial \ln Z/X_0} \frac{\text{d} Z/X_0}{Z/X_0} + \frac{\partial \ln T_{\text{eff}}}{\partial \ln t} \frac{\text{d} t}{t} + \frac{\partial \ln T_{\text{eff}}}{\partial \ln Y_0} \frac{\text{d} Y_0}{Y_0},
\]

\[ \frac{\text{d} \Delta \nu}{\Delta \nu} = \frac{\partial \ln \Delta \nu}{\partial \ln Z/X_0} \frac{\text{d} Z/X_0}{Z/X_0} + \frac{\partial \ln \Delta \nu}{\partial \ln t} \frac{\text{d} t}{t} + \frac{\partial \ln \Delta \nu}{\partial \ln Y_0} \frac{\text{d} Y_0}{Y_0},
\]

where \( \frac{\text{d} M}{M}, \frac{\text{d} t}{t}, \frac{\text{d} Y_0}{Y_0} \) are the unknowns and \( \frac{\text{d} \nu_{\text{max}}}{\nu_{\text{max}}}, \frac{\text{d} \Delta \nu}{\Delta \nu}, \frac{\text{d} T_{\text{eff}}}{T_{\text{eff}}}, \frac{\text{d} Z/X_0}{Z/X_0} \) are the seismic and non-seismic differential constraints.
### Results

**Table:** Relative differences between the two stars, from Eqs. (3)-(5)

<table>
<thead>
<tr>
<th>solar mixture</th>
<th>GN93</th>
<th>AGS05</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Relative Differences of Observational Results</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d\Delta \nu/\Delta \nu \pm \sigma_{\Delta \nu}$</td>
<td>$-0.004 \pm 0.007$</td>
<td></td>
</tr>
<tr>
<td>$dT_{\text{eff}}/T_{\text{eff}} \pm \sigma_{T_{\text{eff}}}$</td>
<td>$0.014 \pm 0.023$</td>
<td></td>
</tr>
<tr>
<td>$d\nu_{\text{max}}/\nu_{\text{max}} \pm \sigma_{\nu_{\text{max}}}$</td>
<td>$0.006 \pm 0.014$</td>
<td></td>
</tr>
<tr>
<td><strong>Relative Differences of Stellar Model Results</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$dM/M \pm \sigma_M$</td>
<td>$0.06 \pm 0.06$</td>
<td>$0.04 \pm 0.05$</td>
</tr>
<tr>
<td>$dt/t \pm \sigma_t$</td>
<td>$-0.33 \pm 0.26$</td>
<td>$-0.24 \pm 0.27$</td>
</tr>
<tr>
<td>$dY_0/Y_0 \pm \sigma_{Y_0}$</td>
<td>$0.03 \pm 0.12$</td>
<td>$0.07 \pm 0.17$</td>
</tr>
<tr>
<td>$dR/R \pm \sigma_R$</td>
<td>$0.02 \pm 0.02$</td>
<td>$0.01 \pm 0.02$</td>
</tr>
</tbody>
</table>
Results

Table: Parameters of HD 175272 obtained by adding the results of the differential analysis with those obtained for HD 181420 for a full computation of adiabatic frequencies.

<table>
<thead>
<tr>
<th>solar mixture</th>
<th>GN93</th>
<th>AGS05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters of Models</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_2$</td>
<td>1.38±0.20</td>
<td>1.33±0.36</td>
</tr>
<tr>
<td>$t_2$(Myr)</td>
<td>1521±271</td>
<td>1829±245</td>
</tr>
<tr>
<td>$(Y_0)_2$</td>
<td>0.31±0.09</td>
<td>0.31±0.16</td>
</tr>
<tr>
<td>Properties of Models</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_2/R_\odot$</td>
<td>1.64</td>
<td>1.65</td>
</tr>
<tr>
<td>$\Delta \nu$</td>
<td>75.01</td>
<td>72.80</td>
</tr>
<tr>
<td>$\nu_{\text{max}}$</td>
<td>1455</td>
<td>1393</td>
</tr>
<tr>
<td>$T_{\text{eff}}$</td>
<td>6655</td>
<td>6645</td>
</tr>
<tr>
<td>$L_2/L_\odot$</td>
<td>4.7</td>
<td>4.8</td>
</tr>
<tr>
<td>log $g_2$</td>
<td>4.15</td>
<td>4.13</td>
</tr>
</tbody>
</table>
The differential analysis method is based on scaling relations, and benefit from the comparison to a star with similar characteristics.

The scientific output of many astroseismic objects with a low SNR benefit from the precise modeling of nearby reference stars with a high SNR.

It can be applied to stars with interesting properties, such as stars hosting an exoplanet or members of a double system.

Perspectives

- Apply the same type of analysis to other types of CoRoT and Kepler’s stars with a low SNR, from red giants to solar-like stars.
- Characterize the well-constrained stars: a very precise determination of the structural differences between nearby stars.
THANKS FOR ATTENTION!